

1       **An Observationally-Based Evaluation of Sub-Grid Scale Ice**  
2       **Thickness Distributions Simulated in a Large-Scale Sea Ice -**  
3       **Ocean Model of the Arctic Ocean**

4                               **Mischa Ungerma<sup>1</sup>, Martin Losch<sup>1</sup>**

5                               <sup>1</sup>Alfred Wegener Institute, Helmholtz Centre for Polar and Marine Research, Bremerhaven, Germany

6       **Key Points:**

- 7       • Recent observations allow to evaluate ice thickness distributions in the Arctic on  
8       regional to local scales.
- 9       • A pan-Arctic model simulates the observed regional and seasonal range of ice  
10      thickness distributions with some skill.
- 11      • The model underestimates the decadal variability of ice thickness distributions.

## Abstract

A key parameterization in sea ice models describes the sub-grid scale ice thickness distribution. Based on only a few observations, the ice thickness distribution model was shown to be consistent with field data and to improve the simulation's large scale properties. The available submarine and airborne observations enable to evaluate in greater detail the ability of a pan-Arctic sea ice - ocean model with an ice thickness distribution parameterization to reproduce observed thickness distributions in different regions and seasons. Many observations are reproduced accurately. Some cases of poorly simulated modes and tails of the distributions are tentatively attributed to simplified thermodynamics and inaccurate deformation fields. Variability on decadal timescales, however, is generally underestimated. Thickness distributions in individual grid cells of the model show similar differences between regions and seasons as observed regional mean distributions, but the modeled grid-scale variability is lower than observed. Simulated modal thicknesses of first-year ice are only insufficiently different from those of multi-year ice. The modal thickness proves to be a useful metric for quantifying model biases in both dynamics and thermodynamics. In addition to improving basin-wide mean variables, the ice thickness distribution parameterization provides reliable and valuable additional sub-grid scale data. At the same time the low climate sensitivity of the parameterization may affect longer simulations with strong climate change aspects.

## 1 Introduction

The Arctic is changing rapidly. Especially the ice cover is in a transition from a perennial to a seasonal state [*Overland et al.*, 2013]. In this situation, evaluating and improving the physical basis of sea ice models becomes increasingly important: (1) climate predictions depend on sea ice models to realistically represent both the feedback processes in the Arctic and the connections between Arctic phenomena and lower latitudes [*Hunke et al.*, 2010]. (2) The reduced sea ice cover sparks economic interest in marine operations like shipping or offshore exploration. Ensuring their safety requires reliable information about the ice cover [*Arctic Council*, 2009].

In this context, small openings in the ice pack, starting from small cracks up to larger leads between floes or linear kinematic features in the ice more than 100 km long, appear as important features whose effect needs to be included in sea ice models. The thin ice in these openings allows for a heat exchange between ocean and atmosphere that

44 is larger by one to two orders of magnitude than the heat exchange over perennial ice  
45 [Maykut, 1978]. For shipping in an ice covered ocean, leads mark divergent regions in  
46 the ice pack and often prescribe the most efficient or only possible routes. These sub-grid  
47 scale features are not wide enough to be fully resolved even in very-high resolution sea ice  
48 models [Hutter *et al.*, 2018] and need to be parameterized.

49 In addition to small openings in the ice, the ice thickness itself varies at the hori-  
50 zontal meter scale. These thickness variations are important for sea ice models, because  
51 the ice growth rate depends inversely, hence non-linearly on ice thickness. Since it is im-  
52 possible to resolve these variations directly, a sub-grid scale Ice Thickness Distribution  
53 (ITD) parameterization was a key element in the first sea ice models [Coon *et al.*, 1974;  
54 Thorndike *et al.*, 1975]. This model component has been adopted in many current climate  
55 models [Stroeve *et al.*, 2014] and has been shown to improve the representation of sea ice  
56 in numerical models [Holland *et al.*, 2006; Massonnet *et al.*, 2011; Komuro and Suzuki,  
57 2013; Ungermann *et al.*, 2017]. Further, an ITD model made possible additional sophisti-  
58 cated parameterizations, for example, a melt pond parameterization [Flocco and Feltham,  
59 2007], or a refined surface stress parameterization [Tsamados *et al.*, 2014].

60 Although this parameterization has been widely used, it was not possible until re-  
61 cently to evaluate the simulated ITDs comprehensively, because not enough reliable ob-  
62 servations were available. Simulated ice thickness distributions in individual grid cells of  
63 early Arctic ITD-enabled models were compared to ice thickness observations from sub-  
64 marines [Hibler, 1980; Flato and Hibler, 1995]. The results were mixed, because only  
65 very few data points were available for comparison and there were large differences be-  
66 tween these individual measurements. Submarine thickness observations were also found  
67 to be too sparse to properly constrain a fully coupled climate model [Bitz *et al.*, 2001],  
68 so that, in extension of the model-observation comparison, the authors focused mostly on  
69 changes between model configurations with and without the ITD parameterization. More  
70 recent evaluations of Arctic ocean sea ice models often used large sets of different obser-  
71 vations to assess the model including observed ITDs, for example, from moorings [Dupont  
72 *et al.*, 2015] or airborne sounding [Herzfeld *et al.*, 2015]. After averaging over multiple  
73 years [Dupont *et al.*, 2015] or over a large region [Herzfeld *et al.*, 2015], the models simu-  
74 lated the observed ITDs accurately.

75 The ITD parameterization has been tested in a Lagrangian sense without being em-  
76 bedded in a dynamic-thermodynamic sea ice model, but forced by observed deformation  
77 and energetic fields. An ITD model of the immediate environment of the drift camp of  
78 the Surface Heat Budget of the Arctic Ocean (SHEBA) experiment was initialized and  
79 forced by sea ice deformation, atmosphere– and ocean state from direct observations,  
80 but the evaluation suffered from the fact that there were no thickness observations be-  
81 yond the initialization phase [*Lindsay, 2003*]. A coastal draft distribution model, forced  
82 with high-precision meteorological observations obtained at the coast, was found to be  
83 largely consistent with draft observations from moorings, but produced excessive ridging  
84 [*Bellchamber-Amundrud et al., 2002*]. A new redistribution model very accurately sim-  
85 ulated observed ice thickness distributions from high-resolution field data in the Gulf of  
86 St. Lawrence, but the simulation and observation period covered only individual strong  
87 deformation events (a storm) over a few days [*Kubat et al., 2010*].

88 In summary, different ITD models have been shown to reproduce different observa-  
89 tions of Arctic ITDs. But at the same time, most authors note that the model results do  
90 not match observations in the generation of open water, or in the amount of very thick ice  
91 produced by ridging, or in the amount of ridging in shearing motion. The most important  
92 processes that form ITDs locally are different in different regions of the Arctic and may  
93 require individual tuning to the local environment. Resulting biases can be reduced based  
94 on individual local observations. Still it is unclear if a pan-Arctic sea ice model that uses  
95 one ITD parameterization with a globally fixed set of parameters can describe different  
96 sea ice regimes accurately.

97 The number of high-resolution sea ice thickness observations has grown steadily  
98 over the past decades. New Airborne ElectroMagnetic (EM) sounding of ice thickness  
99 [*Haas et al., 2010*] complement the Upward-Looking Sonar (ULS) measurements from  
100 submarine cruises [*Rothrock and Wensnahan, 2007*], and detailed evaluations of ice thick-  
101 ness distributions become finally possible. We use this much larger, and until recently  
102 unavailable, database and investigate the extent to which ITD parameterizations can re-  
103 produce regional, seasonal and decadal variability in Arctic ITDs. In the evaluation of the  
104 model results, we focus on three aspects: (1) Does the model reproduce regional averages  
105 of observed distributions? (2) Does the model reproduce single observations at the grid  
106 scale? And (3) which mechanisms and model parameters have the highest impact on the  
107 modeled ITDs? The data set we use and a description of the ITD model are presented

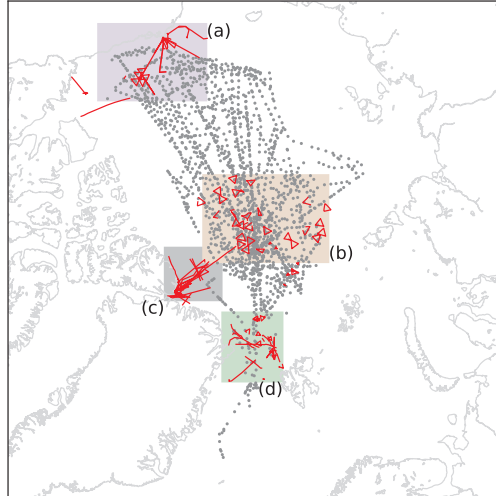
108 in section 2. Model – observation comparisons and the results of additional sensitivity  
 109 studies are presented in section 3. These results are discussed in section 4, and the main  
 110 conclusions are drawn in section 5.

## 111 **2 Methods**

### 112 **2.1 Observations**

116 Since 1958, submarines sailing under the Arctic sea ice have been equipped with  
 117 Upward-Looking Sonar (ULS) that measure the draft of the sea ice. A recent collection of  
 118 submarine-based ULS data and draft distributions for 50km segments of submarine tracks  
 119 covers a large part of the Arctic Ocean and spans the years 1975 to 2005 [*Lindsay and*  
 120 *Schweiger, 2013*]. Airborne electromagnetic (EM) sounding measurements of combined  
 121 ice and snow thickness [e.g. *Haas et al., 2008, 2010*] complement this ULS data set of the  
 122 last 15 years. The lengths of the individual flight tracks during those campaigns differ, but  
 123 they are also in the order of 50km. In this study, we select a subset of these observations  
 124 in four regions (1) Beaufort Sea, (2) Lincoln Sea, (3) Fram Strait and (4) Central Arctic  
 125 (Figure 1). In each of these regions measurement campaigns collected data in similar peri-  
 126 ods of multiple years, so that we can calculate regional mean thickness distributions. Note  
 127 that these averages may not be representative of actual sea ice conditions. It is possible  
 128 that distributions are calculated from the ensemble of observations of extremely different  
 129 ice conditions in different years. Nevertheless, these distributions over a larger sample size  
 130 of comparable forcing conditions allow to test model performance without the need to re-  
 131 produce individual weather events. The sampled observations cover different seasons and  
 132 different decades. The ULS data selected for this study are from the years 1986 – 1997  
 133 and the EM data are from the years 2001 – 2012. Table 1 summarizes the exact years and  
 134 seasons of the observational data sets.

135 ULS and EM soundings can determine the thickness of undeformed ice with high  
 136 accuracy, but they have known biases for ridged ice. The ULS data tend to overestimate  
 137 the thickness by  $29 \text{ cm} \pm 25 \text{ cm}$  [*Rothrock and Wensnahan, 2007*]. One important source  
 138 of error is that the sensors record the fastest reflection of the emitted acoustic signal, so  
 139 that instead of the mean draft, the maximal draft over the footprint of the sensor is ob-  
 140 served. Especially for rough, strongly deformed ice, this leads to an overestimated ice  
 141 draft. The uncertainties in the EM data are as low as 10 cm for level ice [*Pfaffling et al.,*



113 **Figure 1.** Overview of available observations: orange lines for EM-Bird flights, gray dots for ULS sub-  
 114 marine track segments. Shaded areas are the model regions for comparison in (a) Beaufort Sea, (b) Central  
 115 Arctic, (c) Lincoln Sea, (d) Fram Strait.

142 2007], but again the thickness of deformed ice is less accurate. In contrast to the ULS  
 143 data, the electromagnetic sounding measures a weighted mean over a large footprint, so  
 144 that the thickness of individual ridges is mostly smoothed out by the surrounding thinner  
 145 ice. Hence, in EM data the thickness of ridges, and consequently the tail of thickness dis-  
 146 tributions, is underestimated [Reid *et al.*, 2006]. The obtained mean ice thickness is too  
 147 low in the presence of ridges, but the size of this bias is difficult to estimate. The foot-  
 148 print of the sensors is between 2.6 m and 6 m for ULS data [Rothrock and Wensnahan,  
 149 2007] and about 45 m to 75 m for EM data [Reid *et al.*, 2006; Johnston and Haas, 2011].

150 When comparing ULS and EM observations to each other, we convert the ice draft  
 151 to combined ice and snow thickness using the time-dependent values for snow thickness  
 152 and snow density of Warren *et al.* [1999] and constant values  $\rho_w = 1027\text{kg/m}^3$  and  $\rho_i =$   
 153  $928\text{kg/m}^3$  for the densities of water and ice [Rothrock *et al.*, 2008]. To visualize ITDs, we  
 154 plot the probability density for both observations and model data. This allows for a direct  
 155 visual comparison of results, even when the bin sizes of the model are variable. Finally,  
 156 with both measurement techniques it is difficult to distinguish thin ice from open water.  
 157 For this reason, areas with open water are excluded from the calculation of ITDs in this  
 158 study.

159

**Table 1.** Overview of observational data sets

	region <sup>1</sup>	years	months	source	# obs <sup>2</sup>	# campaigns <sup>3</sup>
(S1)	Beaufort Sea	1986–1994	Apr	ULS	32	6
(S2)	Beaufort Sea	1993–1997	Sep, Oct	ULS	54	4
(S3)	Central Arctic	1989–1997	Sep	ULS	117	6
(S4)	Central Arctic	1986–1994	Apr, May	ULS	202	14
(S5)	Fram Strait	1987–1991	Apr, May	ULS	42	2
(S6)	Beaufort Sea	2007–2011	Apr	EM	25	7
(S7)	Lincoln Sea	2004–2012	Apr, May	EM	30	9
(S8)	Central Arctic	2001–2011	Aug, Sep	EM	37	3
(S9)	Fram Strait	2004–2011	Aug	EM	15	3
(S10)	Fram Strait	2003–2011	Apr, May	EM	12	4

<sup>1</sup> Regions as defined in Figure 1

<sup>2</sup> submarine track segments / individual EM-flights

<sup>3</sup> submarine cruises / EM measurement campaigns

160

## 2.2 Model Equations

161

162

163

164

165

166

167

168

169

170

We use the Massachusetts Institute of Technology general circulation model (MIT-gcm, version checkpoint 66a) [Marshall *et al.*, 1997; MITgcm Group, 2016] for our numerical experiments. The model region is the northern cap of a cubed-sphere geometry with an average grid resolution of 36km and boundaries at roughly 55° north in both the Atlantic and the Pacific Ocean [Nguyen *et al.*, 2011]. The necessary boundary conditions are taken from the Estimating the Circulation and Climate of the Ocean, Phase II project (ECCO2) [Menemenlis *et al.*, 2008]. The NCEP Climate Forecast System Reanalysis is used as atmospheric forcing [Saha *et al.*, 2010]. The sea ice component of the MITgcm [Losch *et al.*, 2010] includes dynamics, zero-layer thermodynamics and a dynamic ITD model following Thorndike *et al.* [1975] and Lipscomb *et al.* [2007].

### 2.2.1 Sea Ice Dynamics

The vector  $\mathbf{u}$  of sea ice velocity is calculated according to the momentum balance

$$m \frac{\partial \mathbf{u}}{\partial t} = -m f_C \mathbf{k} \times \mathbf{u} + \boldsymbol{\tau}_a + \boldsymbol{\tau}_o - m \hat{g} \Delta_H + \nabla \cdot \boldsymbol{\sigma}, \quad (1)$$

where  $m = \rho_i H_i + \rho_s H_s$  is the ice and snow mass per unit area, calculated from the respective densities  $\rho_i$ ,  $\rho_s$  and grid cell area averaged thicknesses  $H_i$ ,  $H_s$  of ice and snow. The forcing terms on the right hand side of (1) are: the horizontal Coriolis force with the Coriolis parameter  $f_C$  and the vertical unit vector  $\mathbf{k}$ ; the interfacial stress between atmosphere and ice  $\boldsymbol{\tau}_a$  and ocean and ice  $\boldsymbol{\tau}_o$ ; the sea surface tilt  $\Delta_H$  with the gravitational acceleration  $\hat{g}$ ; and the divergence of the internal ice stress tensor  $\boldsymbol{\sigma}$ . The stresses from atmosphere and ocean on the ice are calculated using the quadratic laws

$$\boldsymbol{\tau}_a = \rho_a c_{d,a} |\mathbf{u}_a - \mathbf{u}| \mathbf{R}_a (\mathbf{u}_a - \mathbf{u}) \quad (2)$$

$$\boldsymbol{\tau}_o = \rho_o c_{d,o} |\mathbf{u}_o - \mathbf{u}| \mathbf{R}_o (\mathbf{u}_o - \mathbf{u}) \quad (3)$$

where  $\rho_a$  and  $\rho_o$  are the reference densities,  $c_{d,a}$  and  $c_{d,o}$  the drag coefficients,  $\mathbf{u}_a$  and  $\mathbf{u}_o$  the velocities, and  $\mathbf{R}_a$  and  $\mathbf{R}_o$  rotation matrices for the atmosphere (subscript  $a$ ) and ocean (subscript  $o$ ) [McPhee, 1975]. More sophisticated parameterizations of drag and drag coefficients in terms of roughness length are available [Tsamados et al., 2014; Roy et al., 2015], that better reflect the complexity of the processes in drag, but for simplicity we employ a commonly used constant drag coefficients formulation. Advection of momentum is neglected in the momentum balance (1), and for simplicity we set the rotation matrices  $\mathbf{R}_a$ ,  $\mathbf{R}_o$  in equations (2) and (3) to unity.

Closing the momentum balance (1) requires a relationship between the stress tensor and the ice drift velocities. We use the standard Reiner-Rivlin constitutive relation for a viscous-plastic rheology [Hibler, 1979] that relates the internal ice stress  $\boldsymbol{\sigma}$  to the strain rate  $\dot{\boldsymbol{\epsilon}} = \frac{1}{2} [\nabla \mathbf{u} + (\nabla \mathbf{u})^T]$ :

$$\boldsymbol{\sigma} = 2\eta \dot{\boldsymbol{\epsilon}} + \left( [\zeta - \eta] \dot{\epsilon}_I - \frac{P}{2} \right) \mathbf{I}. \quad (4)$$

Here the bulk viscosity  $\zeta = \frac{P}{2\Delta_\epsilon}$  and the shear viscosity  $\eta = \frac{\zeta}{e^2}$  are calculated from the ice pressure  $P$ , the axis ratio  $e$  of the elliptical yield curve, and the strain rate tensor  $\dot{\boldsymbol{\epsilon}}$  invariants, that is, divergence  $\dot{\epsilon}_I = \dot{\epsilon}_{11} + \dot{\epsilon}_{22}$  and shear  $\dot{\epsilon}_{II} = \sqrt{(\dot{\epsilon}_{11} - \dot{\epsilon}_{22})^2 + 4\dot{\epsilon}_{12}^2}$ .  $\mathbf{I}$  is the identity matrix and  $\Delta_\epsilon = \sqrt{\dot{\epsilon}_I^2 + e^{-2}\dot{\epsilon}_{II}^2}$  is a convenient measure of deformation specific to the elliptical yield curve. The compressive strength is related to the ice thickness  $h$  and



204 sea ice fractional area  $A$  [Hibler, 1979] through:

$$205 \quad P = P^* A h e^{-C^*(1-A)}. \quad (5)$$

206

207 We briefly introduce three parameterizations of sub-grid scale processes whose im-  
208 pact on modeled ITDs will be investigated later: (1) a gross closing rate  $R_c$  of the ice  
209 pack is calculated as

$$210 \quad R_c = \text{convergence} + C_s * \text{shear} \quad (6)$$

211

212 where convergence =  $-\min(\dot{\epsilon}_I, 0)$  and shear =  $\frac{1}{2}(\Delta\dot{\epsilon} - \text{abs}(\dot{\epsilon}_I))$ . The factor  $0 \leq C_s \leq$   
213 1 determines how much of the shearing motion of the ice pack can be translated to a  
214 closing motion of differently aligned leads, which then ridges ice after the lead is closed  
215 [Flato and Hibler, 1995]. (2) A lead closing parameter  $H_0$  determines how much of newly  
216 formed ice volume is distributed laterally in open water [Hibler, 1979]. The lead closing  
217 parameterization was introduced in 2-category models [Hibler, 1979] to efficiently sum-  
218 marize many small-scale processes during ice formation. With a smaller value, open wa-  
219 ter freezes more quickly and inhibits further heat flux. With a larger value of  $H_0$ , leads  
220 stay open longer which eventually leads to more ice volume in the simulation. In our ITD  
221 model, we apply this parameterization only to the thinnest ice category. This treatment  
222 of new ice is slightly different to other ITD-enabled models [e.g., Lipscomb et al., 2007],  
223 where new ice fills the thinnest category uniformly first and then is ridged into thicker cat-  
224 egories. And (3) during ridging, a fraction of snow  $(1 - F_S)$  with  $0 \leq F_S \leq 1$  is pushed  
225 into the water [Flato and Hibler, 1995].

### 226 **2.2.2 Ice Thickness Distribution**

227 The thickness distribution  $g(h)$  describes the relative amount of ice with thicknesses  
228 between  $h$  and  $h + dh$  [Thorndike et al., 1975]. This distribution can change by advection,  
229 thermodynamics or through ridging. In our simulations, the mechanical changes due to  
230 ridging are parameterized following Thorndike et al. [1975] and Lipscomb et al. [2007]. In  
231 this theory, the horizontal ice motion determines how much ice ridges due to convergence  
232 and shear. When ridging takes place, ice with the distribution  $a(h)$  deforms. This distri-  
233 bution consists mostly of the available thin ice. Ice of initial thickness  $h_{\text{in}}$  is ridged into a  
234 distribution  $\gamma(h_{\text{in}}, h)$ , so that the new ice created by ridging has the thickness distribution

$$235 \quad n(h) = \int_0^{h_{\text{max}}} a(h_{\text{in}}) \gamma(h_{\text{in}}, h) dh_{\text{in}} \quad (7)$$

236

237 for an initial ice cover with a maximal thickness  $h_{\max}$ .

238 We use smooth and differentiable participation and redistribution functions [Lip-  
239 *scomb et al.*, 2007]. The participation function

$$240 \quad a(h) = \frac{1}{b_0} \exp\left(\frac{-G(h)}{a^*}\right) g(h) \quad (8)$$

241  
242 determines how much of the ice of thickness  $h$  takes part in each ridging event.  $b_0$  is a  
243 normalization factor,  $G(h) = \int_0^h g(\hat{h}) d\hat{h}$  is the cumulative thickness distribution and  $a^*$  is  
244 the participation parameter that scales the relative participation of thin and thick ice. And  
245 the redistribution function

$$246 \quad \gamma(h_{\text{in}}, h) = \gamma_0 \exp\left(\frac{-(h - h_{\min})}{\mu\sqrt{h_{\text{in}}}}\right) \quad (9)$$

247  
248 describes how much ice is ridged into thickness  $h$  with each ridged unit area of ice of  
249 thickness  $h_{\text{in}}$ .  $\gamma_0$  is a normalization factor,  $\mu$  is a scaling parameter, and  $h_{\min} = h_{\min}(h_{\text{in}})$   
250 is the minimal thickness into which ice of thickness  $h_{\text{in}}$  can be ridged.

### 251 **2.2.3 Sensitivity Analysis**

252 Sensitivities of the simulated ITDs to ten different parameters are inferred from the  
253 differences between a positive and a negative perturbation run for each parameter. As a  
254 measure of distance between two histograms, we calculate the area between the cumulative  
255 thickness distributions

$$256 \quad d_{\text{hist}}(g_1, g_2) = \int_0^{h_{\max}} |G_1(h) - G_2(h)| dh, \quad (10)$$

257  
258 so that a larger area denotes larger differences between the distributions. This measure is  
259 known as the "Earth mover's Distance": for piles of earth (hence the name), this measure  
260 calculates the minimal amount of work that is necessary to transform one distribution into  
261 the other [Rubner et al., 2000]. With this measure, histograms in different bins can easily  
262 be compared and cross-bin similarities are taken into account.

263 Sensitivities of the simulated ITDs to ten different parameters are inferred from per-  
264 turbation runs. For each parameter, two simulations are performed with a positive and a  
265 negative perturbation; the parameter ranges are given in Table 2. For the mean ITDs in  
266 the regions defined in Table 1 and Figure 1, the mean difference  $d_{\text{hist}}$  between the ITDs  
267 from the two perturbed simulations is used to indicate the sensitivity of the modeled ITDs  
268 to this parameter.

**Table 2.** Parameter values in sensitivity analyzes and final configuration

	Description	Baseline	Perturbation Range	Final
$\mu$ ( $\text{m}^{\frac{1}{2}}$ )	redistribution	3.029	2.029 – 4.029	2.0
$a^*$	participation	0.041	0.031 – 0.051	0.03
$P^*$ ( $\text{kN m}^{-2}$ )	ice strength	22.99	20.0 – 27.0	22.0
$C^*$	strength parameter	15.92	12.0 – 20.0	10.0
$c_{d,a} \times 10^3$	atmospheric drag	1.657	1.4 – 1.9	1.9
$c_{d,o} \times 10^3$	oceanic drag	6.647	6.147 – 7.147	6.5
$e$	axis ratio of ellipse	1.523	1.123 – 1.923	1.8
$C_s$	ridging in shear	0.5	0.25 – 0.75	0.85
$H_0$ (m)	ice growth	0.565	0.415 – 0.715	0.6
$F_s$	snow fraction in ridging	0.5	0.25 – 0.75	0.6

270 The tested parameters are listed in Table 2. They are: the two redistribution param-  
 271 eters (1)  $a^*$ , that determines which ice takes part in ridging processes and (2)  $\mu$ , that de-  
 272 termines the shape of the produced ridges; (3) the compressive ice strength parameter  $P^*$   
 273 and (4) the ice concentration parameter  $C^*$ , of the ice strength parameterization; the drag  
 274 coefficients (5)  $c_{d,a}$  and (6)  $c_{d,o}$  for the ice with respect to atmosphere and ocean; (7) the  
 275 axis ratio  $e$  of the elliptical yield curve, which determines the ratio between shear strength  
 276 and compressive strength  $P$  in the VP-rheology; (8) the shear coefficient  $C_s$ , which deter-  
 277 mines how much energy is used to build pressure ridges in shear deformation; (9) the lead  
 278 closing parameter  $H_0$ ; and (10) the snow fraction  $F_s$  that remains on the ice after ridging.

### 279 2.3 Model Data

280 An Arctic configuration of the MITgcm is compared against the observational data.  
 281 The model setup in this study is based on previous Arctic configurations using the ITD  
 282 parameterization and the Hibler-type strength (5) [Ungermann *et al.*, 2017]. The ITD is  
 283 discretized into ten thickness categories with the bounds 0.0m, 0.32m, 0.66m, 1.04m,  
 284 1.47m, 2.01m, 2.74m, 3.78m, 5.36m, 7.74m. This configuration was chosen as a com-  
 285 promise between computational costs and sufficient thickness resolution. The sensitivity  
 286 analysis informed a manual adjustment of the parameters. The final values, which were

287 chosen so that they improve the representation of the ITD in the model without departing  
 288 too far from the mean sea ice state of the configurations in *Ungermann et al.* [2017], are  
 289 also summarized in Table 2. After a five-year spinup with periodic forcing, the model is  
 290 integrated over the years 1979 to 2011.

291 Model results are compared to observations of either ice draft (ULS) or combined  
 292 ice and snow thickness (EM). In the MITgcm, the ice draft

$$293 \quad h_d = \frac{\rho_i h_i + \rho_s h_s}{\rho_w} \quad (11)$$

294 and the total ice and snow thickness

$$296 \quad h_t = h_i + h_s \quad (12)$$

297 can be calculated from the thicknesses  $h_i$ ,  $h_s$  and densities  $\rho_i$ ,  $\rho_s$  of ice and snow (sub-  
 298 scripts  $i$  and  $s$ ) and the surface density  $\rho_w$  of the ocean.

300 The data coverage allows to assess both regionally averaged ITDs and individual  
 301 measurements. The modeled thickness distributions are averaged over the regions and  
 302 months of the year defined in Table 1 to be compared to the corresponding averages of  
 303 the observations. Track segments of the ULS data and individual flights of the EM data  
 304 are compared to ten-day model snapshots. Each data set is associated with the nearest grid  
 305 cell and the appropriate ten-day snapshot.

### 306 **3 Results**

#### 307 **3.1 Modeled Sea Ice Climate**

308 A quadratic cost function measures the overall performance of our model configu-  
 309 ration. The difference between model results and satellite observations is calculated for  
 310 each point, weighted by the individual measurement uncertainties, and then the squared  
 311 weighted differences are summed. This quantitative indicator of model quality can be  
 312 computed for each satellite product. For more details, the reader is referred to *Ungermann*  
 313 *et al.* [2017]. In our case, the differences to satellite observations of sea ice concentra-  
 314 tion [*EUMETSAT Ocean and Sea Ice Satellite Application Facility*, 2011], sea ice thick-  
 315 ness [*Kwok and Cunningham*, 2008] and sea ice drift during winter and summer months  
 316 [*Lavergne et al.*, 2010; *Kimura et al.*, 2013] weighted by measurement uncertainties are  
 317 evaluated in this way. For ice concentration the uncertainties are provided with the data,  
 318 for ice thickness they are taken as the minimum of 40% of the data value and 1 m, and for

319 ice drift they are constructed from comparisons of different drift data sets [Sumata *et al.*,  
 320 2014, 2015]. The cost function contribution is normalized by the number of individual  
 321 observations for each satellite separately, so that variables with different numbers of data  
 322 points have the same weight in the total cost function (the sum of all contributions). The  
 323 contribution of an individual satellite product is one when the average of the model-data  
 324 misfit in each point is as large as the corresponding measurement uncertainty. In addition,  
 325 the sum of the distances  $d_{\text{hist}}$  between model results and observations for the ten regions  
 326 defined in Table 1 is calculated as a measure of the overall quality of the modeled ITDs.

328 The cost function terms and the quantitative ITD comparisons are summarized in  
 329 Table 3 for the configuration used in this study and two configurations from *Ungermann*  
 330 *et al.* [2017]: the best configuration of the latter study “ITD5H” with with five thickness  
 331 categories and a Hibler-type ice strength parameterization and a reference configuration  
 332 “noITD” without an dynamic ITD parameterization. This comparison is not completely  
 333 unbiased, because the measure  $d_{\text{hist}}$  depends slightly on the resolution of the ITD, and  
 334 because the ITDs for the configuration “noITD” are calculated from mean thickness per  
 335 grid cell in the respective regions only. Still, the combination of the results shows that the  
 336 tuning described in Section 2.2.3 permits a better representation of the ITDs. These im-  
 337 provements are mainly obtained in the comparisons with the EM data. Some Arctic-wide  
 338 sea ice features, such as concentration and winter drift, cannot be improved by tuning the  
 339 model to ITD, and their cost function contributions increase. The overall model quality  
 340 with the adjusted parameters as measured by the cost function, however, is comparable to  
 341 a well-tuned configuration without an active ITD.

342 In addition, we compare model results for Arctic-wide sea ice volume and extent to  
 343 the results from the Pan-Arctic Ice Ocean Modeling and Assimilation System [PIOMAS,  
 344 *Schweiger et al.*, 2011] and to observations from the Sea Ice Index [*Windnagel et al.*, 2016]  
 345 in Figure 2. For both variables the model simulates a seasonal cycle with the same tim-  
 346 ing as in the reference data, but a slightly lower magnitude. Note that the model does not  
 347 show the unrealistically high seasonality that is expected of models using 0-layer thermo-  
 348 dynamics [*Semtner*, 1984]. The model tends to underestimate the sea ice volume and at  
 349 the same time overestimate the extent, indicating that the modeled mean ice thickness is  
 350 too low. For both variables, the linear trend over the three decades is clearly lower in the  
 351 model than in the reference data, independent of the different signs in model bias. The  
 352 year-to-year variability in the observations is captured for extent, but for volume there are

327

**Table 3.** Cost Function Values and  $d_{\text{hist}}$  for Regional ITDs

	present study	ITD5H (2017) <sup>1</sup>	noITD (2017) <sup>2</sup>
concentration	1.75	1.57	1.69
thickness	0.63	0.63	0.75
winter drift	0.65	0.45	0.5
summer drift	0.9	0.95	1.03
<b>Total cost function</b>	<b>3.94</b>	<b>3.59</b>	<b>3.97</b>
$d_{\text{hist}}$ EM	2.92	4.19	4.26
$d_{\text{hist}}$ ULS	5.26	5.54	7.15
<b>Total <math>d_{\text{hist}}</math></b>	<b>8.18</b>	<b>9.73</b>	<b>11.41</b>

<sup>1</sup> ITD with 5 categories + Hibler-type strength [Ungermann *et al.*, 2017]

<sup>2</sup> Two-category thickness model [Ungermann *et al.*, 2017]

353

larger differences between consecutive years in the PIOMAS model than in our MITgcm

354

simulation.

358

### 3.2 Regional Ice Thickness Distributions

362

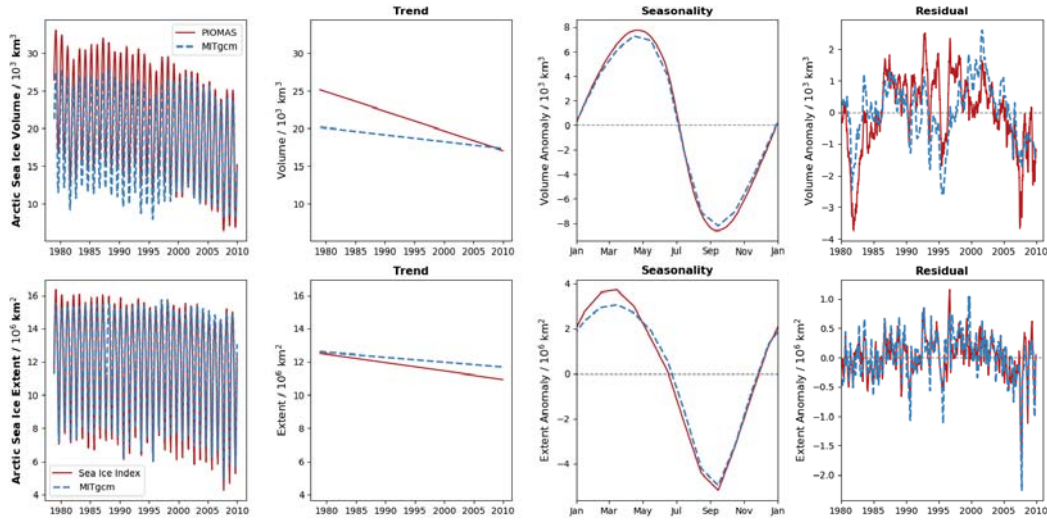
We first compare the model simulation to the ten data sets of Table 1 (Figure 3).

363

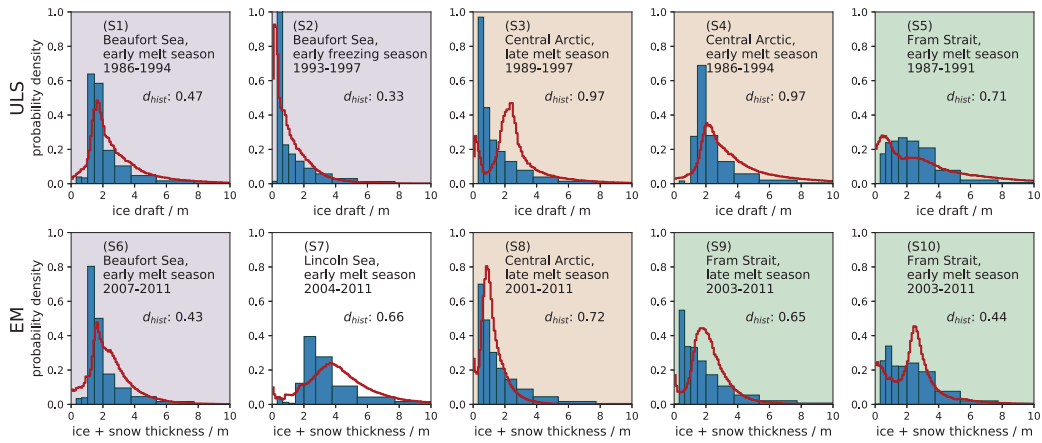
364

Similar to the observations, the modeled ice thickness distributions vary between regions, but there are differences in the accuracy of the modeled ice conditions that appear to depend on the ice type. In predominantly first-year ice in the Beaufort Sea and in the Central Arctic during the 2000s, the agreement between model and observations is very high (Figure 3, (S1), (S2), (S6), and (S8)). For example, the integrated differences  $d_{\text{hist}}$  between the cumulative histograms of observations and model distributions of 0.33m, 0.43m and 0.47m are much lower in the Beaufort Sea than the average over all regions of  $0.64\text{m} \pm 0.21\text{m}$ . In regions with more multi-year ice, the model still captures the overall properties of the ice pack, but especially for bi-modal distributions (Figure 3, (S3), (S9), and (S10)), the agreement with the observed ITDs is lower. The seasonal variations in the ITDs are also best represented with first-year ice: The model slightly underestimates the changes in the Beaufort Sea (S1 vs. S2), while it strongly overestimates the annual cycle in the Central Arctic (S3 vs. S4) and in the Fram Strait (S9 vs. S10).

375



355 **Figure 2.** Comparison of the MITgcm against PIOMAS (Arctic Sea Ice Volume)  
 356 (Arctic Sea Ice Extent). The time series are separated into a linear trend, a seasonal fluctuation and the resid-  
 357 ual that is not explained by the two.



359 **Figure 3.** Regional ITDs from model (blue bars) and observations (red line). Observations are ice draft  
 360 from submarine ULS (S1)–(S5), ice + snow thickness from airborne EM sounding (S6) – (S10). Exact re-  
 361 gions and times of comparisons are specified in Figure 1 and Table 1.

376 The model does not simulate large decadal differences in ITDs. We compare average  
 377 distributions of combined model ice and snow thickness centered at 1990 and 2005 for re-  
 378 gions and seasons in analogy to the observations S1, S2, S3, S4, S5, S7 and S9 (see Table  
 379 1). On average, the modal thicknesses of the model distributions do not change over these  
 380 years ( $0\text{m} \pm 0.02\text{m}$ ). In this evaluation, S5 is excluded because the distribution is very flat  
 381 around its mode. Over the same time, the mean thicknesses of the modeled distributions  
 382 decrease only by  $0.06\text{m} \pm 0.12\text{m}$ . In comparison, for the three regions with observations in  
 383 different decades, the estimated loss in mean ice and snow thickness is 0.88m (S1 and S6,  
 384 Beaufort Sea), 1.79m (S3 and S8, Central Arctic) and 1.37m (S5 and S10, Fram Strait).

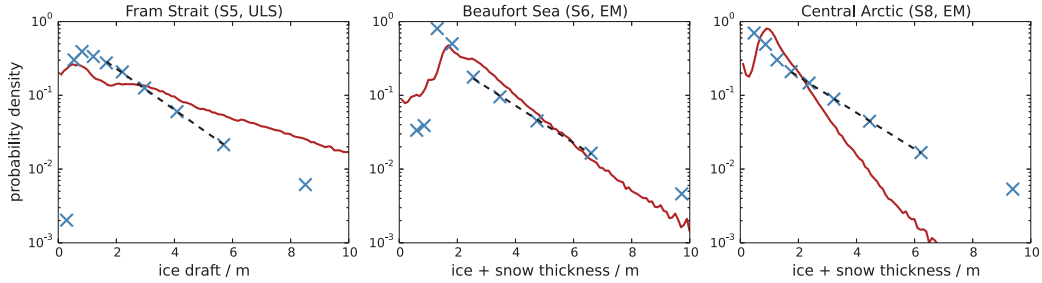
385 The model underestimates both modal and mean thickness compared to observa-  
 386 tions, but the differences are smaller for the mean than for the mode. On average, the  
 387 modal thicknesses of the ten regions are thinner by  $0.66\text{m} \pm 0.89\text{m}$  in the model than in  
 388 the observations, while the difference for the mean thicknesses is only  $0.25\text{m} \pm 0.47\text{m}$ ,  
 389 indicating that the distributions in the model are skewed compared to the observations:  
 390 While the mode in the model is often unrealistically thin and introduces too much thin ice  
 391 into the distribution, its effect on the mean thickness is partially offset by too much ridged  
 392 ice and too little ice that is thinner than the mode.

396 The exponential tails in the distributions further illustrate these differences in the  
 397 shape of the ITDs. Both the observed and the modeled ITDs show an exponential tail, but  
 398 the rate parameters (or the slopes in the semi-logarithmic plot) are different (Figure 4).  
 399 While the qualitative behavior of the tail agrees between model and observations, the rate  
 400 parameter of the modeled tail differs in most regions from the observations. Of the three  
 401 regions in Figure 4, the distribution tails of model and observations agree in the Beau-  
 402 fort Sea mostly because the observed distribution is very different from other regions. The  
 403 thickness distributions in Figure 4 were chosen to represent the range of distribution tails  
 404 in the model and the observations. In other regions the distributions decay with compara-  
 405 ble exponential tails.

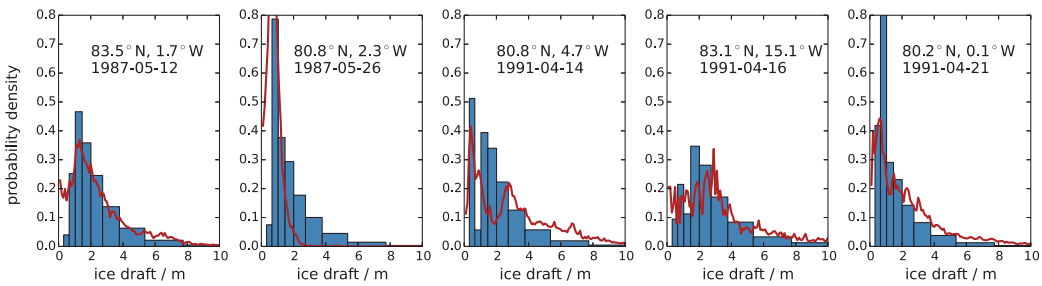
### 406 3.3 Grid-Scale Ice Thickness Distributions

410 Draft distributions from five different submarine track segments are compared to the  
 411 distributions taken from the nearest model grid cell (Figure 5). We note, that this com-  
 412 parison is more sensitive to biases in the ITD parameterizations than the comparison of

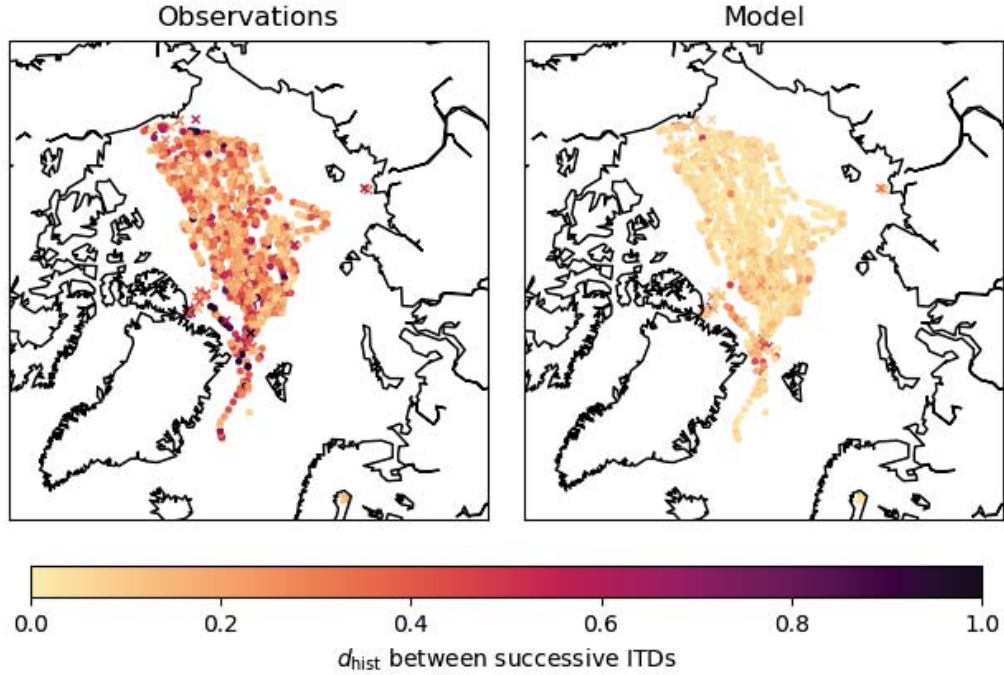




393 **Figure 4.** Semi-logarithmic plot of average ice draft  $h_d$  or ice and snow thickness  $h_t$  against probability  
 394 density in each category for three regional ITD. Blue crosses for model values, red lines for observations. The  
 395 dashed black lines indicate exponential fits to the model results.



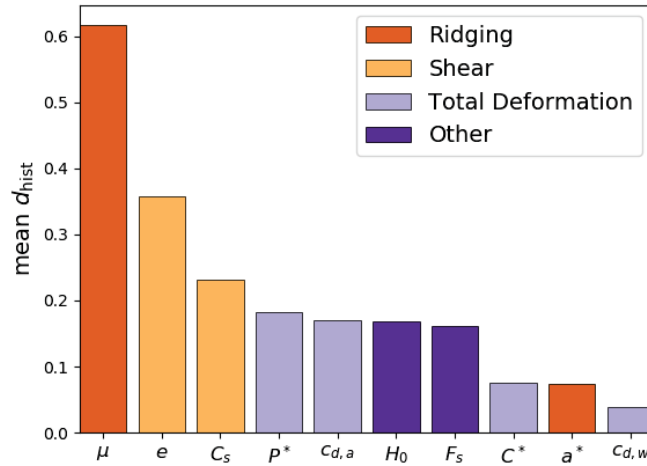
407 **Figure 5.** Example of variability in ITDs on small local scales. Plotted are ITDs from 50km submarine  
 408 track segments (red line) with a snapshot from the nearest grid cell (blue bars). All five observations are taken  
 409 in Fram Strait in spring (S5).



418 **Figure 6.** Integrated differences  $d_{\text{hist}}$  between the cumulative ITDs for successive 50 km track segments  
 419 (ULS) / successive flights (EM) (left hand side) and between the corresponding model snapshots (right hand  
 420 side) as a measure of grid-scale variability. Circles mark ULS observations, crosses mark EM observations.

413 regional mean values because it does not involve any smoothing by averaging. The choice  
 414 of observations represents the range of locally observed thickness distributions and their  
 415 simulation in the model. The model distributions follow the main characteristics of the  
 416 observations. The variability of the local thickness distributions in a given region is gener-  
 417 ally smaller for the model than for the observations.

421 We quantify the local variability by calculating the integrated difference  $d_{\text{hist}}$  be-  
 422 tween the cumulative thickness distributions for consecutive 50km track segments of ULS  
 423 campaigns and for consecutive flights in EM sounding campaigns (Figure 6). All succes-  
 424 sive pairs of observations in the individual campaigns are evaluated except for pairs that  
 425 are more than 200 km apart. This provides a good coverage of the central half of the Arc-  
 426 tic Ocean, although there are more data points in the older ULS data set (1131) than in  
 427 the more recent EM data set (62). For both data sets, this measure of variability is clearly



431 **Figure 7.** Sensitivity of modeled regional ITDs to each parameter. The difference  $d_{\text{hist}}$  between the re-  
 432 gional ITDs of the positive and negative perturbation simulations is calculated, the mean result is plotted. The  
 433 color coding refers to different physical mechanisms.

428 higher for the observations ( $0.30 \text{ m} \pm 0.26 \text{ m}$  for ULS and  $0.40 \text{ m} \pm 0.37 \text{ m}$  for EM) than for  
 429 the model ( $0.06 \text{ m} \pm 0.07 \text{ m}$  for ULS and  $0.14 \text{ m} \pm 0.13 \text{ m}$  for EM).

### 430 3.4 Sensitivity Studies

434 Sensitivity experiments show the relative impact of ten parameters (Table 2) on the  
 435 modeled ITDs. The redistribution during ridging (parameter  $\mu$ ) and the deformation in  
 436 shear (parameters  $e$  and  $C_s$ ) are most important in shaping the modeled ITDs. Figure 7  
 437 summarizes the sensitivity of the regional ITDs to the full set of parameters.

438 Adjusting the ridging parameterization, especially the redistribution of thicknesses  
 439 during ridging ( $\mu$ ), leads to the largest changes in the modeled ITD. Note that adjusting  
 440 the participation function (parameter  $a^*$ ) leaves the ITDs nearly unchanged. The sensitiv-  
 441 ity of the ITDs to both  $e$  and  $C_s$  is still larger than the sensitivity to  $P^*$  or  $c_{d,a}$ . This is an  
 442 interesting result because the ice strength  $P^*$  and the atmospheric drag coefficient  $c_{d,a}$  are  
 443 among the most commonly used parameters for tuning sea ice models towards large-scale  
 444 observations [Nguyen *et al.*, 2011]. Our results suggest that they are not the first choice for  
 445 tuning regional ITDs.

#### 446 **4 Discussion**

447 The model with ITD parameterization simulates regional and seasonal differences  
 448 in ITDs accurately compared to corresponding observations. To our best knowledge, we  
 449 show with unprecedented detail that an ITD model not only simulates average ice con-  
 450 ditions in one region accurately [e.g. *Dupont et al.*, 2015; *Herzfeld et al.*, 2015], but also  
 451 successfully simulates very different regional and seasonal ITDs with the same configu-  
 452 ration and parameter set. The model tends to produce distributions with a thin peak and  
 453 an exponentially decaying tail of thicker, ridged ice. On the one hand, this leads to small  
 454 model-observation misfits for conditions of relatively uniform first-year ice, for example, in  
 455 the Beaufort Sea. On the other hand, bi-modal distributions with multiple ice types, which  
 456 are common for example in Fram Strait, are poorly represented.

457 The modeled ITDs do not change very much over 15 years. We calculated a re-  
 458 duction of mean ice and snow thickness from the observations for a similar time span.  
 459 However, the ULS data are collected before 1997 and generally overestimate mean ice  
 460 thickness, while the EM data are collected after 2001 and underestimate mean ice thick-  
 461 ness, so that the computed difference in mean thickness probably overestimates the ac-  
 462 tual changes. *Rothrock et al.* [2008] found the Arctic wide mean sea ice draft to have de-  
 463 creased by 0.54 m over 15 years. The underlying data set was far more comprehensive  
 464 than ours, which allowed to reduce the bias between ULS and EM data. Still, the reduc-  
 465 tion of 0.54 m is bigger than even the largest ice thickness reduction in our model.

466 Our simulated grid-cell ITDs reproduce mean conditions, but underestimate the vari-  
 467 ability between points that are in close proximity. Previous comparisons of ITDs from sin-  
 468 gle grid cells of an Arctic model [*Hibler*, 1980] or of results of single-column ITD mod-  
 469 els [*Schramm et al.*, 1997; *Bellchamber-Amundrud et al.*, 2002] to the few then available  
 470 point-wise observations agree with ours in that sea ice simulations with the ITD parame-  
 471 terization are consistent with observed Arctic ITDs and that the parameterization can be  
 472 tuned to a specific set of observations. But with the currently available data, we can go  
 473 further to show that the parameterization simulates ITDs in single grid cells that are very  
 474 similar to the regional mean states, but underestimate the observed variability between  
 475 neighboring grid cells.

476 In the following subsections, we discuss two possible sources of differences between  
 477 model and observations: the sea ice thermodynamics and the sea ice deformation. A de-

478 tailed evaluation of modeled ITDs can help to discriminate between these processes as a  
479 source of error.

#### 480 **4.1 Thermodynamics**

481 Modeled ITDs are not very different between different decades. Both the Arctic-  
482 wide decadal trends in sea ice extent and in sea ice volume are low in the model when  
483 compared to satellite observations or other, well-validated model results. In the following  
484 we speculate that the simple 0-layer thermodynamics of our model is one reason for the  
485 low interdecadal variability.

486 The 0-layer thermodynamics were derived to provide a simple, cost-efficient, and  
487 easy to implement way to calculate thermodynamic fluxes through the ice [*Semtner, 1976*]  
488 at the cost of reduced physical realism. With this parameterization, sea ice does not pos-  
489 sess any internal heat capacity, so that the ice warms instantaneously and melting pro-  
490 cesses start as soon as the air temperatures rise above the freezing point in spring. The  
491 consequence is a phase error of approximately one month and a tendency to overestimate  
492 the seasonal sea ice thickness cycle [*Semtner, 1984*]. These systematic biases can be re-  
493 duced by adjusting other sea ice parameters, especially albedo and sea ice conductivity to  
494 adjust ice growth and melt rates, and ridging parameters to arrive at realistic mean sea ice  
495 thicknesses for unrealistic growth rates [*Semtner, 1984*].

496 In spite of the simple thermodynamics, the seasonal cycle in the model configuration  
497 of this study matches the reference data very closely for pan-Arctic integral properties.  
498 This is so because most parameter values in this study are based on results from previ-  
499 ous model optimizations against large sets of different observations [*Nguyen et al., 2011*;  
500 *Ungermann et al., 2017*] to reduce the typical biases associated with 0-layer thermodynam-  
501 ics. It is reasonable to think that this parameter optimization led to some sort of overfit-  
502 ting: We assume that the choice of parameters does not describe the underlying physical  
503 processes faithfully, but leads to a nonlinear combination of effects that produces the good  
504 match to observations. Such an overfitting to the limited time span of reliable observations  
505 may explain why both climate sensitivities and long-term changes in the ITDs are under-  
506 estimated in the model.

507 We find that our model underestimates mean ice thickness to a smaller degree than  
508 modal ice thickness. The modal thickness in Arctic ITDs is a standard diagnostic in eval-

509 uations of high-resolution ice thickness observations [e.g. *Haas and Howell, 2015*], be-  
 510 cause it describes mostly undeformed, thermodynamically grown ice. In contrast, sea ice  
 511 models are often evaluated against mean sea ice thickness [e.g. *Chevallier et al., 2016*;  
 512 *Stroeve et al., 2014*]. Again, tuning the models towards this target can introduce com-  
 513 pensating biases in the thermodynamics and the ridging schemes: in most regional ITDs  
 514 our model simulates a mode that is too thin compared to observations; this mode is com-  
 515 pensated by a heavier tail leading to a mean thickness that hides the model deficit. This  
 516 indicates weaker thermodynamic ice growth and compensating ridge distributions in the  
 517 model compared to observations. *Herzfeld et al. [2015]* simulated ice draft distributions  
 518 with characteristics in the Fram Strait that are similar to ours (Section 3) even though they  
 519 use a model with much more sophisticated thermodynamics. From this we speculate that  
 520 compensating biases exist also in more sophisticated models, and that an evaluation of the  
 521 modal thickness can help to identify some of them.

522 In passing we note that the reported effects of the 0-layer thermodynamics, espe-  
 523 cially the rapid onset and high melt rates in fall, may explain why there are no bi-modal  
 524 ITDs in the model. In every spring and summer, ice melts too rapidly, so that ice surviv-  
 525 ing the melting season is not sufficiently thick to produce a distinct, thicker second mode,  
 526 or to create a first mode as thick as in the observations. In spite of the inaccurate physics  
 527 on the small scale that lead to missing modes in the ITDs, we do not observe the unrealis-  
 528 tically strong seasonal cycle for ice thickness that is expected for 0-layer thermodynamics.  
 529 A more detailed examination of these processes on a local scale is beyond the scope of  
 530 this study.

## 531 **4.2 Sea Ice Deformation**

532 The redistribution of ice thickness due to ridging depends directly on the deforma-  
 533 tion field (equation 6). A realistic model representation of sea ice deformation in shear  
 534 [*Bouchat and Tremblay, 2017*; *Wang et al., 2016*; *Kwok and Cunningham, 2016*] and the  
 535 scaling of sea ice deformation on small scales [*Weiss and Dansereau, 2017*; *Oikkonen*  
 536 *et al., 2017*; *Spreen et al., 2017*; *Hutter et al., 2018*] are current research topics. In this  
 537 section we show that both deformation in shear and the localization of deformation are  
 538 inherently connected to the simulation of ITDs.

539 We find that the exponential decay of the tail of the simulated distributions is indica-  
 540 tive of an appropriate model of the physics of ridging. The exponentially decreasing tail  
 541 of thick ice is a common feature of observed Arctic ice thickness distributions [*Wadhams*  
 542 *and Davy*, 1986]. Similar tails are simulated with different ITD models with constant re-  
 543 distribution [*Bellchamber-Amundrud et al.*, 2002; *Godlovitch et al.*, 2012] suggesting that  
 544 the exponential tails are not created by the explicitly exponential redistribution functions  
 545 used in this study. Instead, their results suggest that the appropriate physical mechanisms  
 546 included in the ITD parameterization are not sensitive to the details of the redistribution  
 547 function. But note that the inaccurately modeled slopes of the exponential distributions  
 548 (Figure 4) in combination with the difficulty of tuning model coefficients to minimize  
 549 competing biases (see section 4.1) indicate that one set of parameters in the redistribution  
 550 functions that has been tuned to fit mean thickness does not necessarily lead to realistic  
 551 ridging behavior in all forcing situations. *Kubat et al.* [2010] tune redistribution schemes  
 552 to reproduce individual deformation events accurately. Choosing a comparable target for  
 553 parameter optimization may reduce possible biases that are introduced to the ridging pa-  
 554 rameterization when tuning it towards mean ice thickness. However, as long as, for exam-  
 555 ple, the consolidation of multi-year ridges is not explicitly included in the model and ridge  
 556 properties are solely determined by the initial ridging process, adjusting the ridging pa-  
 557 rameters (such as  $\mu$  in this study) can change the ridge geometries to be more in line with  
 558 first-year or multi-year ice [*Lipscomb et al.*, 2007]. Especially with the strong reduction of  
 559 the multi-year ice fraction over the last decades [*Polyakov et al.*, 2012], this may limit the  
 560 ability of current redistribution schemes to reproduce shifts in the ITDs.

561 The sensitivity studies emphasize how important deformation properties in shear are  
 562 for sea ice models. Both shear parameters  $e$  and  $C_s$  are used in many current sea ice mod-  
 563 els with their default values. These values of  $e = 2$  [*Hibler*, 1979] and  $C_s = 0.5$  [*Flato*  
 564 *and Hibler*, 1995] are not very well constrained by observations. For example, decreas-  
 565 ing the value of  $e$  can improve the representation of different Arctic-wide sea ice features  
 566 [*Miller et al.*, 2005; *Lemieux et al.*, 2016; *Bouchat and Tremblay*, 2017; *Ungermann et al.*,  
 567 2017]. Furthermore, a recent study analyzed deformation fields and thickness changes  
 568 from coinciding satellite observations and suggested that the majority of mechanical ice  
 569 thickness redistribution is caused by shear instead of convergence [*Kwok and Cunningham*,  
 570 2016]. Our results support the notion that deformation in shear is a key factor in shaping  
 571 different ITDs in the Arctic and that stronger ridging caused by shear (i.e., a larger value

572 of  $C_s = 0.85$ ) is required to improve the model simulations with respect to observed ITDs.  
 573 At the same time, we note that with a redistribution scheme tuned towards direct obser-  
 574 vations of deformation events as in *Kubat et al.* [2010] it should be possible to constrain  
 575 uncertain shear parameters such as  $e$  with the increasing number of regional ITD observa-  
 576 tions.

577 Finally, low- and medium-resolution models with a viscous-plastic rheology and  
 578 smooth and slow atmospheric forcing are known to produce unrealistically smooth defor-  
 579 mation fields [*Girard et al.*, 2009]. This agrees with our observations of low grid-scale  
 580 variability in the ITDs: Since the deformation fields with our grid resolution of 36km  
 581 have only very few sharp features, the deformation history of neighboring grid cells should  
 582 be very similar. Increasing the resolution of similar VP models allows to reach a realistic  
 583 degree of localization of deformation, even though the intermittency continues to be un-  
 584 derestimated [*Hutter et al.*, 2018]. We note that in our comparison of 1D observations to  
 585 2D model results, the 2D results are already smoother by representing an average of 2  
 586 dimensions. Still, from future high-resolution simulations of model configurations with  
 587 a dynamic ITD we may learn if the appropriate localization of deformation is more im-  
 588 portant than tuning shear deformation parameters of ITDs or vice versa. New rheologies  
 589 may simulate the observed intermittency of deformation more realistically than the VP-  
 590 rheology. Comparing different rheologies in high-resolution simulations may then provide  
 591 insight into whether localization of deformation is more important than its intermittency  
 592 in the simulation of ITDs. In addition, comparing different rheologies in high-resolution  
 593 simulations may inform about the role of grid-scale ITDs in generating intermittency of  
 594 deformation.

## 595 **5 Conclusions**

596 From a comparison of modeled ITDs against observations from different regions,  
 597 seasons and decades in the Arctic, we draw the following conclusions: With the currently  
 598 used form of ITD parameterizations one can accurately reproduce many but not all ice  
 599 thickness distributions under different forcing situations in the Arctic in the same sim-  
 600 ulation. Observed regional and seasonal variations in ITDs in the Arctic are, to a large  
 601 degree, reproduced both in regional averages and snapshots from single grid cells. Indi-  
 602 vidual regional ITDs have been modeled successfully before, but here we show a general



603 agreement in a pan-Arctic sea ice - ocean model. Decadal variations in ITDs, however, are  
604 lower than observed

605 The modeled ITDs depend on the overall drift and thickness patterns and hence on  
606 parameters that are not directly related to the ITD parameterization. In some cases the  
607 model does not capture the mode and tail of observed ice thickness distributions. There  
608 are many potential reasons for this and we have tried to attribute some of these issues to  
609 simplified thermodynamics and inaccurate deformation fields. The attribution is not com-  
610 plete and it is difficult to disentangle all different process that are involved.

611 With the many new high resolution thickness data, we presented the shape of ITDs,  
612 and especially their modal thickness, as new, and easy to implement model diagnostics.  
613 The modal thickness is a key parameter in evaluating observations, and we suggest that it  
614 should also be used in evaluating model results. The modal thickness diagnostic allows to  
615 separate more clearly thermodynamic and dynamic effects in thickness patterns, and can  
616 thereby reduce potentially compensating biases in these two parameterizations.

617 Variability in ITDs between adjacent grid points is low in the model. The parame-  
618 terization should be local by design, yet the simulated ITDs in individual grid cells react  
619 mostly to regional conditions. We identify different possible causes, with smooth deforma-  
620 tion fields in medium resolution VP models as the most probable one. Future studies with  
621 dynamic ITD parameterizations in high resolution models can identify if improved local-  
622 ization of deformation will improve grid cell ITDs or if low intermittency in deformation  
623 is a limiting factor.

## 624 **Acknowledgments**

625 We would like to thank Martin Vancoppenolle and Fred Dupont for their helpful reviews  
626 on this paper. Observational data are taken from [*Lindsay and Schweiger, 2013*] (ULS)  
627 and [*Haas et al., 2006, 2008, 2010*] (EM). Model data will be made available at PAN-  
628 GAEA upon acceptance of the paper. This project was supported by the Deutsche Forschungs-  
629 gemeinschaft (DFG) through the International Research Training Group IRTG 1904 Arc-  
630 Train.

## 631 **References**

632 Arctic Council (2009), Arctic Marine Shipping Assessment 2009 Report, *Tech. rep.*

- 633 Bellchamber-Amundrud, T., H. Melling, and G. Ingram (2002), Modelling the Evolution  
634 of Draft Distribution in the Sea Ice Pack of the Beaufort Sea, in *Ice in the Environment:  
635 Proceedings of the 16th IAHR International Symposium on Ice*, December.
- 636 Bitz, C. M., M. M. Holland, A. J. Weaver, and M. Eby (2001), Simulating the ice-  
637 thickness distribution in a coupled climate model, *Journal of Geophysical Research*,  
638 *106*, 2441, doi:10.1029/1999JC000113.
- 639 Bouchat, A., and B. Tremblay (2017), Using sea-ice deformation fields to constrain the  
640 mechanical strength parameters of geophysical sea ice, *Journal of Geophysical Research:  
641 Oceans*, *122*(7), 5802–5825, doi:10.1002/2017JC013020.
- 642 Chevallier, M., G. C. Smith, F. Dupont, J.-F. Lemieux, G. Forget, Y. Fujii, F. Hernandez,  
643 R. Msadek, K. A. Peterson, A. Storto, T. Toyoda, M. Valdivieso, G. Vernieres, H. Zuo,  
644 M. Balmaseda, Y.-S. Chang, N. Ferry, G. Garric, K. Haines, S. Keeley, R. M. Kovach,  
645 T. Kuragano, S. Masina, Y. Tang, H. Tsujino, and X. Wang (2016), Intercomparison of  
646 the Arctic sea ice cover in global ocean–sea ice reanalyses from the ORA-IP project,  
647 *Climate Dynamics*, *SI*, doi:10.1007/s00382-016-2985-y.
- 648 Coon, M. D., G. Maykut, R. S. Pritchard, D. A. Rothrock, and A. S. Thorndike (1974),  
649 Modeling the Pack Ice as an Elastic-Plastic Material, *AIDJEX Bulletin*, *24*, 1–106.
- 650 Dupont, F., S. Higginson, R. Bourdallé-Badie, Y. Lu, F. Roy, G. C. Smith, J.-F. Lemieux,  
651 G. Garric, and F. Davidson (2015), A high-resolution ocean and sea-ice modelling sys-  
652 tem for the Arctic and North Atlantic oceans, *Geoscientific Model Development*, *8*(5),  
653 1577–1594, doi:10.5194/gmd-8-1577-2015.
- 654 EUMETSAT Ocean and Sea Ice Satellite Application Facility (2011), Global sea ice con-  
655 centration reprocessing dataset 1978-2009 (v1.1), [Online]. Norwegian and Danish Me-  
656 teorological Institutes. Available from <http://osisaf.met.no>.
- 657 Flato, G. M., and W. D. Hibler (1995), Ridging and strength in modeling the thickness  
658 distribution of Arctic sea ice, *Journal of Geophysical Research*, *100*(C9), 18,611–18,626.
- 659 Flocco, D., and D. L. Feltham (2007), A continuum model of melt pond evolution on Arc-  
660 tic sea ice, *Journal of Geophysical Research*, *112*(C8), doi:10.1029/2006JC003836.
- 661 Girard, L., J. Weiss, J. M. Molines, B. Barnier, and S. Bouillon (2009), Evaluation of  
662 high-resolution sea ice models on the basis of statistical and scaling properties of Arctic  
663 sea ice drift and deformation, *Journal of Geophysical Research: Oceans*, *114*(8), 1–15,  
664 doi:10.1029/2008JC005182.

- 665 Godlovitch, D., A. Monahan, and G. Flato (2012), An idealised stochastic model of  
666 sea ice thickness dynamics, *Cold Regions Science and Technology*, 78, 14–30, doi:  
667 10.1016/j.coldregions.2012.02.006.
- 668 Haas, C., and S. E. L. Howell (2015), Ice thickness in the Northwest Passage, *Geophysical*  
669 *Research Letters*, 42(18), 7673–7680, doi:10.1002/2015GL065704.
- 670 Haas, C., S. Hendricks, and M. Doble (2006), Comparison of the sea ice thickness dis-  
671 tribution in the Lincoln Sea and adjacent Arctic Ocean in 2004 and 2005, *Annals of*  
672 *Glaciology*, 44, 247–252.
- 673 Haas, C., A. Pfaffling, S. Hendricks, L. Rabenstein, J. L. Etienne, and I. Rigor (2008),  
674 Reduced ice thickness in Arctic Transpolar Drift favors rapid ice retreat, *Geophysical*  
675 *Research Letters*, 35(17), 1–5, doi:10.1029/2008GL034457.
- 676 Haas, C., S. Hendricks, H. Eicken, and A. Herber (2010), Synoptic airborne thickness sur-  
677 veys reveal state of Arctic sea ice cover, *Geophysical Research Letters*, 37(9), 1–5, doi:  
678 10.1029/2010GL042652.
- 679 Herzfeld, U. C., E. C. Hunke, B. W. McDonald, and B. F. Wallin (2015), Sea ice deforma-  
680 tion in Fram Strait – Comparison of CICE simulations with analysis and classification  
681 of airborne remote-sensing data, *Cold Regions Science and Technology*, 117, 19–33, doi:  
682 10.1016/j.coldregions.2015.05.001.
- 683 Hibler, W. D. (1979), A Dynamic Thermodynamic Sea Ice Model, *Jour-*  
684 *nal of Physical Oceanography*, 9(4), 815–846, doi:10.1175/1520-  
685 0485(1979)009<0815:ADTSIM>2.0.CO;2.
- 686 Hibler, W. D. (1980), Modeling a Variable Thickness Sea Ice Cover, *Monthly Weather Re-*  
687 *view*, 108(12), 1943–1973.
- 688 Holland, M. M., C. M. Bitz, E. C. Hunke, W. H. Lipscomb, and J. L. Schramm (2006),  
689 Influence of the sea ice thickness distribution on polar climate in CCSM3, *Journal of*  
690 *Climate*, 19(11), 2398–2414, doi:10.1175/JCLI3751.1.
- 691 Hunke, E. C., W. H. Lipscomb, and A. K. Turner (2010), Sea-ice models for climate  
692 study: retrospective and new directions, *Journal of Glaciology*, 56(200), 1162–1172,  
693 doi:10.3189/002214311796406095.
- 694 Hutter, N., M. Losch, and D. Menemenlis (2018), Scaling Properties of Arctic Sea  
695 Ice Deformation in a High-Resolution Viscous-Plastic Sea Ice Model and in Satel-  
696 lite Observations, *Journal of Geophysical Research: Oceans*, 123(1), 672–687, doi:  
697 10.1002/2017JC013119.

- 698 Johnston, M. E., and C. Haas (2011), Validating Helicopter-based EM (HEM) Thicknesses  
699 Over Very Thick Multi-Year Ice, in *Proceedings of the 21st International Conference on*  
700 *Port and Ocean Engineering under Arctic Conditions (POAC '11)*, pp. 1–11, International  
701 Conference on Port and Ocean Engineering under Arctic Conditions, Montréal, Canada.
- 702 Kimura, N., A. Nishimura, Y. Tanaka, and H. Yamaguchi (2013), Influence of win-  
703 ter sea-ice motion on summer ice cover in the Arctic, *Polar Research*, 32, 1–8, doi:  
704 10.3402/polar.v32i0.20193.
- 705 Komuro, Y., and T. Suzuki (2013), Impact of subgrid-scale ice thickness distri-  
706 bution on heat flux on and through sea ice, *Ocean Modelling*, 71, 13–25, doi:  
707 10.1016/j.ocemod.2012.08.004.
- 708 Kubat, I., M. Sayed, S. B. Savage, and T. Carrieres (2010), Numerical simulations of ice  
709 thickness redistribution in the Gulf of St. Lawrence, *Cold Regions Science and Technol-*  
710 *ogy*, 60(1), 15–28, doi:10.1016/j.coldregions.2009.07.003.
- 711 Kwok, R., and G. F. Cunningham (2008), ICESat over Arctic sea ice: Estimation of  
712 snow depth and ice thickness, *Journal of Geophysical Research*, 113(C8), C08,010, doi:  
713 10.1029/2008JC004753.
- 714 Kwok, R., and G. F. Cunningham (2016), Contributions of growth and deformation  
715 to monthly variability in sea ice thickness north of the coasts of Greenland and the  
716 Canadian Arctic Archipelago, *Geophysical Research Letters*, 43(15), 8097–8105, doi:  
717 10.1002/2016GL069333.
- 718 Lavergne, T., S. Eastwood, Z. Teffah, H. Schyberg, and L.-A. Breivik (2010), Sea ice  
719 motion from low-resolution satellite sensors: An alternative method and its val-  
720 idation in the Arctic, *Journal of Geophysical Research*, 115(C10), C10,032, doi:  
721 10.1029/2009JC005958.
- 722 Lemieux, J.-F., C. Beaudoin, F. Dupont, F. Roy, G. C. Smith, A. Shlyueva, M. Buehner,  
723 A. Caya, J. Chen, T. Carrieres, L. Pogson, P. Derpentigny, A. Plante, P. Pestieau,  
724 P. Pellerin, H. Ritchie, G. Garric, and N. Ferry (2016), The Regional Ice Prediction  
725 System (RIPS): Verification of forecast sea ice concentration, *Quarterly Journal of the*  
726 *Royal Meteorological Society*, 142(695), 632–643, doi:10.1002/qj.2526.
- 727 Lindsay, R. (2003), Changes in the modeled ice thickness distribution near the Surface  
728 Heat Budget of the Arctic Ocean (SHEBA) drifting ice camp, *Journal of Geophysical*  
729 *Research*, 108(C6), doi:10.1029/2001JC000805.

- 730 Lindsay, R., and A. J. Schweiger (2013, updated 2017), Unified Sea Ice Thickness Climate  
731 Data Record, 1947 onward, Version 1, doi:<http://dx.doi.org/10.7265/N5D50JXV>.
- 732 Lipscomb, W. H., E. C. Hunke, W. Maslowski, and J. Jakacki (2007), Ridging, strength,  
733 and stability in high-resolution sea ice models, *Journal of Geophysical Research:*  
734 *Oceans*, *112*, 1–18, doi:10.1029/2005JC003355.
- 735 Losch, M., D. Menemenlis, J. M. Campin, P. Heimbach, and C. Hill (2010), On the for-  
736 mulation of sea-ice models. Part 1: Effects of different solver implementations and pa-  
737 rameterizations, *Ocean Modelling*, *33*(1-2), 129–144, doi:10.1016/j.ocemod.2009.12.008.
- 738 Marshall, J., A. Adcroft, C. Hill, L. Perelman, and C. Heisey (1997), A finite-volume,  
739 ncompressible Navier Stokes model for studies of the ocean on parallel computers,  
740 *Journal of Geophysical Research*, *102*(C3), 5753–5766, doi:10.1029/96JC02775.
- 741 Massonnet, F., T. Fichefet, H. Goosse, M. Vancoppenolle, P. Mathiot, and C. König Beatty  
742 (2011), On the influence of model physics on simulations of Arctic and Antarctic sea  
743 ice, *The Cryosphere*, *5*(3), 687–699, doi:10.5194/tc-5-687-2011.
- 744 Maykut, G. A. (1978), Energy exchange over young sea ice in the central Arctic, *Journal*  
745 *of Geophysical Research*, *83*(C7), 3646, doi:10.1029/JC083iC07p03646.
- 746 McPhee, M. (1975), Ice-Momentum Transfer for the AIDJEX Ice Model, *AIDJEX Bulletin*,  
747 *29*, 93–112.
- 748 Menemenlis, D., J-M. Campin, P. Heimbach, C. Hill, T. Lee, A. Nguyen, M. Schodlok,  
749 and H. Zhang (2008), ECCO2: High resolution global ocean and sea ice data synthesis,  
750 *Mercator Ocean Quarterly Newsletter*, *31*(October), 13–21.
- 751 Miller, P. A., S. W. Laxon, and D. L. Feltham (2005), Improving the spatial distribu-  
752 tion of modeled Arctic sea ice thickness, *Geophysical Research Letters*, *32*(18), doi:  
753 10.1029/2005GL023622.
- 754 MITgcm Group (2016), MITgcm Manual.
- 755 Nguyen, A. T., D. Menemenlis, and R. Kwok (2011), Arctic ice-ocean simulation with op-  
756 timized model parameters: Approach and assessment, *Journal of Geophysical Research:*  
757 *Oceans*, *116*(4), 1–18, doi:10.1029/2010JC006573.
- 758 Oikkonen, A., J. Haapala, M. Lensu, J. Karvonen, and P. Itkin (2017), Small-scale sea ice  
759 deformation during N-ICE2015: From compact pack ice to marginal ice zone, *Journal*  
760 *of Geophysical Research: Oceans*, *122*(6), 5105–5120, doi:10.1002/2016JC012387.
- 761 Overland, J. E., M. Wang, J. E. Walsh, and J. C. Stroeve (2013), Future Arctic cli-  
762 mate changes : Adaptation and mitigation time scales, *Earth's Future*, *2*, 68–74, doi:

- 763 10.1002/2013EF000162.Received.
- 764 Pfaffling, A., C. Haas, and J. E. Reid (2007), Direct helicopter EM  $\tilde{\sim}$  Sea-ice thickness  
765 inversion assessed with synthetic and field data, *Geophysics*, 72(4), F127–F137, doi:  
766 10.1190/1.2732551.
- 767 Polyakov, I. V., J. E. Walsh, and R. Kwok (2012), Recent changes of Arctic multiyear  
768 sea ice coverage and the likely causes, *Bulletin of the American Meteorological Society*,  
769 93(2), 145–151, doi:10.1175/BAMS-D-11-00070.1.
- 770 Reid, J. E., A. Pfaffling, and J. Vrbancich (2006), Airborne electromagnetic footprints in  
771 1D earths, *Geophysics*, 71(2), 63–72, doi:10.1190/1.2187756.
- 772 Rothrock, D. A., and M. Wensnahan (2007), The accuracy of sea ice drafts measured from  
773 U.S. Navy submarines, *Journal of Atmospheric and Oceanic Technology*, 24(11), 1936–  
774 1949, doi:10.1175/JTECH2097.1.
- 775 Rothrock, D. A., D. B. Percival, and M. Wensnahan (2008), The decline in arctic sea-ice  
776 thickness: Separating the spatial, annual, and interannual variability in a quarter cen-  
777 tury of submarine data, *Journal of Geophysical Research: Oceans*, 113(5), 1–9, doi:  
778 10.1029/2007JC004252.
- 779 Roy, F., M. Chevallier, G. C. Smith, F. Dupont, G. Garric, J.-F. Lemieux, Y. Lu, and  
780 F. Davidson (2015), Arctic sea ice and freshwater sensitivity to the treatment of the  
781 atmosphere-ice-ocean surface layer, *Journal of Geophysical Research: Oceans*, 120,  
782 4392–4417, doi:10.1002/2014JC010677.
- 783 Rubner, Y., C. Tomasi, and L. J. Guibas (2000), The Earth Mover’s Distance as a Met-  
784 ric for Image Retrieval, *International Journal of Computer Vision*, 40(2), 99–121, doi:  
785 10.1023/A:1026543900054.
- 786 Saha, S., S. Moorthi, H. L. Pan, X. Wu, J. Wang, S. Nadiga, P. Tripp, R. Kistler,  
787 J. Woollen, D. Behringer, H. Liu, D. Stokes, R. Grumbine, G. Gayno, J. Wang, Y. T.  
788 Hou, H. Y. Chuang, H. M. H. Juang, J. Sela, M. Iredell, R. Treadon, D. Kleist, P. Van  
789 Delst, D. Keyser, J. Derber, M. Ek, J. Meng, H. Wei, R. Yang, S. Lord, H. Van Den  
790 Dool, A. Kumar, W. Wang, C. Long, M. Chelliah, Y. Xue, B. Huang, J. K. Schemm,  
791 W. Ebisuzaki, R. Lin, P. Xie, M. Chen, S. Zhou, W. Higgins, C. Z. Zou, Q. Liu,  
792 Y. Chen, Y. Han, L. Cucurull, R. W. Reynolds, G. Rutledge, and M. Goldberg (2010),  
793 The NCEP climate forecast system reanalysis, *Bulletin of the American Meteorological*  
794 *Society*, 91(August), 1015–1057, doi:10.1175/2010BAMS3001.1.

- 795 Schramm, J. L., M. M. Holland, and J. A. Curry (1997), The effects of snowfall on a  
796 snow-ice-thickness distribution, *Annals of Glaciology*, 25(25), 287–291.
- 797 Schweiger, A., R. Lindsay, J. Zhang, M. Steele, H. Stern, and R. Kwok (2011), Un-  
798 certainty in modeled Arctic sea ice volume, *Journal of Geophysical Research*,  
799 116(September), doi:10.1029/2011JC007084.
- 800 Semtner, A. J. (1984), On modelling the seasonal thermodynamic cycle of sea ice in stud-  
801 ies of climatic change, *Climatic Change*, 6(1), 27–37, doi:10.1007/BF00141666.
- 802 Semtner, A. J. J. (1976), A model for the thermodynamic growth of sea ice in numerical  
803 investigations of climate, *Journal of Physical Oceanography*, 6, 379–389.
- 804 Spreen, G., R. Kwok, D. Menemenlis, and A. T. Nguyen (2017), Sea-ice deformation in a  
805 coupled ocean–sea-ice model and in satellite remote sensing data, *The Cryosphere*,  
806 11(4), 1553–1573, doi:10.5194/tc-11-1553-2017.
- 807 Stroeve, J. C., A. P. Barrett, M. C. Serreze, and A. Schweiger (2014), Using records from  
808 submarine, aircraft and satellite to evaluate climate model simulations of Arctic sea ice  
809 thickness, *The Cryosphere*, 8, 2179–2212, doi:10.5194/tcd-8-2179-2014.
- 810 Sumata, H., T. Lavergne, F. Girard-Ardhuin, N. Kimura, M. A. Tschudi, F. Kauker,  
811 M. Karcher, and R. Gerdes (2014), An intercomparison of Arctic ice drift products  
812 to deduce uncertainty estimates, *Journal of Geophysical Research: Oceans*, 119, doi:  
813 10.1002/2013JC009724.
- 814 Sumata, H., R. Kwok, R. Gerdes, F. Kauker, and M. Karcher (2015), Uncertainty of Arc-  
815 tic summer ice drift assessed by high-resolution SAR data, *Journal of Geophysical Re-  
816 search: Oceans*, 120, 5285–5301, doi:10.1002/2014JC010632.
- 817 Thorndike, A. S., D. A. Rothrock, G. A. Maykut, and R. Colony (1975), The Thickness  
818 Distribution of Sea Ice, *Journal of Geophysical Research*, 80(33), 4501–4513, doi:  
819 10.1029/JC080i033p04501.
- 820 Tsamados, M., D. L. Feltham, D. Schroeder, D. Flocco, S. L. Farrell, N. Kurtz, S. W.  
821 Laxon, and S. Bacon (2014), Impact of Variable Atmospheric and Oceanic Form Drag  
822 on Simulations of Arctic Sea Ice, *Journal of Physical Oceanography*, 44(5), 1329–1353,  
823 doi:10.1175/JPO-D-13-0215.1.
- 824 Ungermann, M., L. B. Tremblay, T. Martin, and M. Losch (2017), Impact of the ice  
825 strength formulation on the performance of a sea ice thickness distribution model  
826 in the Arctic, *Journal of Geophysical Research: Oceans*, 122, 2017–2033, doi:  
827 10.1002/2016JC012128.

- 828 Wadhams, P., and T. Davy (1986), On the spacing and draft distributions for  
829 pressure ridge keels, *Journal of Geophysical Research*, *91*(C9), 10,697, doi:  
830 10.1029/JC091iC09p10697.
- 831 Wang, Q., S. Danilov, T. Jung, L. Kaleschke, and A. Wernecke (2016), Sea ice leads in  
832 the Arctic Ocean: Model assessment, interannual variability and trends, *Geophysical*  
833 *Research Letters*, *43*(13), 7019–7027, doi:10.1002/2016GL068696.
- 834 Warren, S. G., I. G. Rigor, N. Untersteiner, V. F. Radionov, N. N. Bryazgin, Y. I. Alek-  
835 sandrov, and R. Colony (1999), Snow depth on Arctic sea ice, *Journal of Climate*, *12*,  
836 1814–1829, doi:10.1175/1520-0442(1999)012<1814:SDOASI>2.0.CO;2.
- 837 Weiss, J., and V. Dansereau (2017), Linking scales in sea ice mechanics, *Philosophical*  
838 *Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences*,  
839 *375*(2086), 20150,352, doi:10.1098/rsta.2015.0352.
- 840 Windnagel, A., M. Savoie, and W. Meier (2016), Sea Ice Index Version 2 Analysis, *Tech.*  
841 *rep.*, National Snow and Ice Data Center, Boulder CO, USA.